

R-modes of neutron stars with a solid crust

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ABSTRACT

We investigate the properties of *r*-mode oscillations of a slowly rotating neutron star with a solid crust, by taking account of the effects of the Coriolis force. For the modal analysis we employ three-component neutron star models that are composed of a fluid core, a solid crust and a surface fluid ocean. For the three-component models, we find that there exist two kinds of *r*-modes, that is, those confined in the surface fluid ocean and those confined in the fluid core, which are most important for the *r*-mode instability. The *r*-modes do not have any appreciable amplitudes in the solid crust if rotation rate of the star is sufficiently small. We find that the core *r*-modes are strongly affected by mode coupling with the crustal torsional (toroidal) modes and lose their simple properties of the eigenfunction and eigenfrequency as functions of the angular rotation velocity Ω . This indicates that the extrapolation formula, which is obtained in the limit of $\Omega \rightarrow 0$, cannot be used to examine the *r*-mode instability of rapidly rotating neutron stars if they have a solid crust.

Subject headings: instabilities — stars: neutron — stars: oscillations — stars: rotation

1. Introduction

Since the discovery of the gravitational radiation driven instability of the *r*-modes by Andersson (1998) and Friedman & Morsink (1998), a large number of studies on the properties of *r*-modes and inertial modes of rotating stars have been done to prove their possible importance in astrophysics (for a recent review see, e.g., Friedman & Lockitch 1999). Although early investigations of the *r*-mode instability were mainly applied to young hot neutron stars (e.g., Lindblom, Owen, & Morsink 1998; Owen et al. 1998), recently some authors have also discussed possible roles of the *r*-mode instability in old and cool neutron stars with a solid crust and a magnetic field, as those found in low mass X-ray binaries (LMXBs) (e.g., Andersson, Kokkotas, & Stergioulas 1999; Bildsten & Ushomirsky 2000; Rezzolla, Lamb, & Shapiro 2000). For example, Bildsten & Ushomirsky (2000) have suggested that the *r*-mode instability can be largely weakened by the effects of viscous damping in the boundary layer at the interface between the solid crust and the fluid core. Rezzolla et al. (2000) have also suggested that the *r*-mode oscillation is quickly damped by emitting electromagnetic radiation from the oscillating magnetic field of the star.

It is well known that neutron stars with a solid crust show a rich spectrum of nonradial oscillation modes, for example, nonradial modes associated with the solid crust and those associated with the fluid core and the surface fluid ocean (McDermott et al. 1988). If the effects of rotation are taken into account, it is quite common to find avoided crossings, as functions of Ω , between various oscillation modes of the

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neutron star model with a solid crust (Lee & Strohmayer 1996). This is also the case for r -modes. When the angular rotation frequency Ω of the star is small, the r -mode frequency observed in the corotating frame may be given by

$$\omega_r(\Omega) \approx \frac{2m\Omega}{l'(l'+1)}, \quad (1)$$

where l' and m are the indices of the spherical harmonic function representing the dominant toroidal component of the displacement vector. On the other hand, the crustal toroidal mode frequency may be given by (e.g., Strohmayer 1991)

$$\omega_t(\Omega) \approx \omega_t(0) + \frac{m\Omega}{l'(l'+1)}, \quad (2)$$

where $\omega_t(0)$ is the oscillation frequency of the mode at $\Omega = 0$. The frequencies of the two modes get close to each other at

$$\Omega_{cross} \approx \frac{l'(l'+1)}{m} \omega_t(0). \quad (3)$$

As shown by McDermott et al. (1988) and Lee & Strohmayer (1996), the oscillation frequency of the crustal toroidal mode that has no radial nodes of the eigenfunction is of order of $\omega_t(0)/\sqrt{GM/R^3} \sim 10^{-2}$ for $l' = 2$, and hence we have $\Omega_{cross}/\sqrt{GM/R^3} \sim 10^{-2} \times (l' + 1)$ for the modes with $l' = m$, where M and R are the mass and radius of the neutron star, and G is the gravitational constant. This suggests that the mode crossing can happen between the two modes at sufficiently small rotation rates. Almost all previous discussions on the r -mode instability, however, did not take account of the effects of mode crossing between the two different kinds of modes. In fact, the discussions have been based on the assumptions that the modal property of the r -modes is almost independent of the rotation frequency Ω and hence that an extrapolation formula, obtained in the limit of $\Omega \rightarrow 0$, is applicable to rapidly rotating neutron stars with $\Omega/\sqrt{GM/R^3} \sim 1$. Obviously these two assumptions are not necessarily correct for the r -modes that experience mode coupling with other kinds of oscillation modes.

In this paper, we examine the r -mode instability of neutron stars that have a solid crust, by considering the effects of the mode crossing between the r -modes and the crustal toroidal modes. The plan of this paper is as follows. In §2, we show the modal properties of the r -modes of neutron stars with a solid crust. In §3, we discuss the dissipation timescales of the r -modes. §4 is for discussions and conclusions.

2. r -Modes of Neutron Stars with a Solid Crust

Neutron star models we use in this paper are the same as those used in the modal analysis by McDermott et al. (1988). The models are taken from the evolutionary sequences for cooling neutron stars calculated by Richardson et al. (1982), where the envelope structure is constructed by following Gudmundsson, Pethick & Epstein (1983). These models are composed of a fluid core, a solid crust and a surface fluid ocean, and the interior temperature is finite and is not constant as a function of the radial distance r . The models are not isentropic and the Schwarzschild discriminant A has finite values in the interior (see, e.g., Yoshida & Lee 2000b).

The method of calculation of nonradial oscillations of rotating neutron stars with a solid crust is the same as that used by Lee & Strohmayer (1996), in which the Cowling approximation is employed. Only the terms due to the Coriolis force are included in the perturbation equations and no effects of the centrifugal force are considered. This approximation may be justified for low frequency modes satisfying $|2\Omega/\omega| \geq 1$ and $\Omega^2/(GM/R^3) \ll 1$, where ω is the oscillation frequency observed in the corotating frame of the star

(Unno et al. 1989). The eigenfunctions are expanded in terms of spherical harmonic functions $Y_l^m(\theta, \varphi)$ with different values of l for a given m . Here spherical polar coordinates (r, θ, φ) are used. For example, the Lagrangian displacement vector, ξ_i is expanded as

$$\xi_r = r \sum_{l \geq |m|}^{\infty} S_l(r) Y_l^m(\theta, \varphi) e^{i\sigma t}, \quad (4)$$

$$\xi_\theta = r \sum_{l, l' \geq |m|}^{\infty} \left\{ H_l(r) \frac{\partial Y_l^m}{\partial \theta} + T_{l'}(r) \frac{1}{\sin \theta} \frac{\partial Y_{l'}^m}{\partial \varphi} \right\} e^{i\sigma t}, \quad (5)$$

$$\xi_\varphi = r \sum_{l, l' \geq |m|}^{\infty} \left\{ H_l(r) \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \varphi} - T_{l'}(r) \frac{\partial Y_{l'}^m}{\partial \theta} \right\} e^{i\sigma t}, \quad (6)$$

where $l = |m| + 2k$ and $l' = l + 1$ for even modes and $l = |m| + 2k + 1$ and $l' = l - 1$ for odd modes where $k = 0, 1, 2, \dots$ (Lee & Strohmayer 1996, see also Lee & Saio 1986). The symbol σ denotes the oscillation frequency observed in an inertial frame, and we have $\omega = \sigma + m\Omega$. In this paper, to obtain a good convergence of the eigenfunctions and eigenfrequencies at a given Ω , we keep an enough number of terms in the series expansion of the eigenfunctions.

We computed frequency spectra of r -modes, inertial modes, and crustal toroidal modes for the three-component neutron star models called NS05T7, NS05T8, and NS13T8 (see, McDermott et al. 1988). The obtained mode spectra for the three models are qualitatively the same. In this paper, we therefore show the mode spectrum for the model NS13T8, because this is the most compact model among the three. The mass M and the radius R of the model NS13T8 are $M = 1.326 M_\odot$ and $R = 7.853$ km, respectively, and the central temperature is $T_c = 1.05 \times 10^8$ K.

In Figure 1, scaled eigenfrequencies $\kappa \equiv \omega/\Omega$ of r -modes, inertial modes, and a crustal toroidal mode of the model NS13T8 are given as functions of $\bar{\Omega} \equiv \Omega/\sqrt{GM/R^3}$ for $m = 2$, where ω is the frequency observed in the corotating frame of the star. Here only the fundamental r -modes with $l' = m = 2$ are considered since they are most important for the r -mode instability of neutron stars (see Lockitch & Friedman 1999; Yoshida & Lee 2000a, 2000b). The crustal toroidal mode shown in Figure 1 is the fundamental mode with no radial nodes of the eigenfunction and has the smallest oscillation frequency of the mode of this kind for a given l' at $\Omega = 0$ (see McDermott et al. 1988). Note that at $\bar{\Omega} \sim 0$ it is practically impossible to correctly calculate rotational modes because of their coupling with high overtone g -modes. In this figure, we have used the notation given by ${}_{l'}r_n^c$ and ${}_{l'}r_n^s$ for the r -modes, ${}_{l'}t_n$ for the crustal toroidal modes, and ${}_{l_0}i$ for the inertial modes, where l' denotes the index of the spherical harmonic function associated with the dominant toroidal component of the displacement vector, n is the number of radial nodes of the eigenfunction, and l_0 is the number introduced to classify the inertial modes (see, e.g., Yoshida & Lee 2000a). Note that the mode classification has been done at sufficiently small values of $\bar{\Omega}$. Since there appear surface r -modes and core r -modes for the three-component neutron star models, we have also introduced the superscripts s and c to distinguish the two kinds of r -modes. The oscillation energy of the core r -mode is predominantly confined in the fluid core, and that of the surface r -mode is confined in the surface ocean. A typical eigenfunction of a surface r -mode is given in Lee & Strohmayer (1996). The modes in the figure are all odd modes (with $l' = |m|$ for the toroidal modes) and they are retrograde modes propagating in the opposite direction to that of stellar rotation.

Figure 1 clearly shows that the mode crossing between the $2r_0^c$ -mode and the $2t_0$ -mode, as predicted by equations (1) to (3), leads to an avoided crossing of their frequencies as functions of $\bar{\Omega}$. At the avoided

crossing, the modal properties of the two modes are exchanged. This is understood by examining the eigenfunctions of the two modes before and after the crossing, which are indicated by the filled circles in Figure 1, where the labels (a), (c), and (d) attaching to the filled circles are corresponding to the panels (a), (c), and (d) in Figure 2. Note that the ${}_2t_0$ -mode at $\bar{\Omega} = 4 \times 10^{-3}$, which should carry the label (b), is not shown in Figure 1, because $\kappa \gg 1$. In Figure 2, the expansion coefficients S_3 , H_3 , and iT_2 of the two modes with $l' = m = 2$ at $\bar{\Omega} = 4 \times 10^{-3}$ in panels (a) and (b) and at $\bar{\Omega} = 4 \times 10^{-2}$ in panels (c) and (d) are given as functions of the fractional radius r/R , where the eigenfunctions are normalized as $iT_{l'}(r_{bc}) = 1$ with r_{bc} being the bottom of the solid crust of the model. When $\bar{\Omega}$ is sufficiently small, the ${}_2r_0^c$ -mode in panel (a) and the ${}_2t_0$ -mode in panel (b) have their typical eigenfunctions and the influence of other modes on the eigenfunction is negligible. Through the avoided crossing, however, the modal properties of the two modes are exchanged along the frequency curves as $\bar{\Omega}$ increases. As shown by Figure 2, the mode in panel (d), which is corresponding to the mode (d) on the frequency curve of ${}_2t_0$ in Figure 1, has in the fluid core the eigenfunction characteristic of the ${}_2r_0^c$ -mode. On the other hand, the mode in panel (c), which corresponds to the mode (c) on the frequency curve of ${}_2r_0^c$, has in the crust the eigenfunction characteristic of the ${}_2t_0$ -mode. However, this avoided crossing between the two modes is not sharp in the sense that the eigenfunction of each of the two modes suffers the contamination of the other mode after the avoided crossing. For example, the mode (d), which may be regarded as a core r -mode, has also oscillation amplitudes in the solid crust. This is in a sharp contrast with avoided crossings, for example, between ${}_2r_0^s$ and ${}_2r_0^c$ and between ${}_5i$ and ${}_2r_0^c$, for which the mode properties are almost completely exchanged through the crossings. Because of the avoided crossing between the ${}_2r_0^c$ -mode and the crustal toroidal modes, the ${}_2r_0^c$ -mode will lose its simple modal property of the eigenfrequency and the eigenfunction as a function of $\bar{\Omega}$. We note that the fundamental r -mode with $l' = m$ of fluid stars has such a simple modal property as a function of $\bar{\Omega}$ (e.g., Yoshida & Lee 2000b; Karino et al. 2000).

Avoided crossing between the ${}_2r_0^c$ and ${}_2t_0$ modes with $l' = m$ may be common for neutron stars with a solid crust. To show this, we calculate the ${}_2r_0^c$ modes with $l' = m = 2$ for $n = 1$ polytropic neutron star models in which a solid crust is artificially embedded. We assume the same physical parameters, such as the mass, radius, and the crust thickness, as those of the model NS13T8. For the shear modulus μ_0 , we assume that the ratio $\mu_0/\rho^{4/3}$ is constant in the crust where ρ is the mass density (see e.g., McDermott et al. 1988). In Table 1, we tabulated the values of $\mu_0/\rho^{4/3}$ in the crust and the frequencies $\omega_t(0)$ of the fundamental ${}_2t_0$ mode at $\bar{\Omega} = 0$. We note that $\omega_t(0)$ of the polytropic model is approximately proportional to the value of $\sqrt{\mu_0/\rho^{4/3}}$. Figure 3 gives the scaled frequencies $\kappa = \omega/\Omega$ of the ${}_2r_0^c$ -mode as functions of $\bar{\Omega}$ for the polytropic models. The figure clearly shows the avoided crossing between the ${}_2r_0^c$ -mode and the ${}_2t_0$ -mode is common and occurs at the values of $\bar{\Omega}_{cross}$ expected from equation (3).

3. Dissipation Timescales of Core r -Modes

For comparison with other studies on the r -mode instability, let us derive an extrapolation formula for the damping timescale of the fundamental core r -mode with $l' = m = 2$ by calculating the r -modes at sufficiently small values of $\bar{\Omega}$, for which the mode coupling with the crustal toroidal mode is negligibly weak. Then, using the same shear and bulk viscosity coefficients as those employed in Yoshida & Lee (2000a) and taking account of the viscous boundary layer (VBL) damping effects following the formulation given in Bildsten & Ushomirsky (2000), we may approximately express the formula as (see e.g., Lindblom et al.

1998)

$$\begin{aligned} \frac{1}{\tau} = & \frac{1}{\tilde{\tau}_S} \left(\frac{10^8 \text{ K}}{T_c} \right)^2 + \frac{1}{\tilde{\tau}_B} \left(\frac{T_c}{10^8 \text{ K}} \right)^6 \left(\frac{\Omega^2}{\pi G \bar{\rho}} \right) \\ & + \frac{1}{\tilde{\tau}_{VBL}} \left(\frac{10^8 \text{ K}}{T_c} \right) \left(\frac{\Omega^2}{\pi G \bar{\rho}} \right)^{1/4} + \frac{1}{\tilde{\tau}_{J,2}} \left(\frac{\Omega^2}{\pi G \bar{\rho}} \right)^3, \end{aligned} \quad (7)$$

where $\bar{\rho}$ is the average density of the star, and T_c is the central temperature. The same viscosity coefficients are used both in the fluid and solid regions, and no effects of superfluidity in the core and in the inner crust are considered. Here, the first, second, third and fourth terms in the right-hand side of equation (7) are contributions from the shear viscosity, the bulk viscosity, the shear viscosity in VBL and the current quadrupole radiation, respectively. The factors containing the ratio $T_c/10^8 \text{ K}$ have been introduced in equation (7) to extrapolate the damping rates with respect to the central temperature T_c .

In Table 2, the various dissipation timescales $\tilde{\tau}$ of the $2r_0^c$ modes with $l' = m = 2$ are tabulated for the model NS13T8 and a simple $n = 1$ polytropic model with a static solid crust. For the polytropic model, we employ $M = 1.4M_\odot$ and $R = 12.5\text{km}$ and assume that a solid crust forms in the density region of $\rho \leq 1.5 \times 10^{14} \text{ g/cm}^3$. Note that no wave propagation is assumed in the static crust of the polytropic model employed in this section. Bildsten & Ushomirsky (2000) (see also Anderson et al. 2000) used the similar polytropic model with a static solid crust to estimate the VBL damping effects on the r -modes. To compute the r -modes of the simple polytropic model, the method given in Yoshida & Lee (2000a), in which effects of the centrifugal force and perturbations of the gravitational potential are taken into account, has been employed. As shown by Table 2, all the dissipative timescales, except that due to the bulk viscosity, have similar values both for the simple polytropic model and for the model NS13T8. We note that although the destabilization due to the current quadrupole radiation for the model NS13T8 is by one order of magnitude stronger than that for the polytropic model, the VBL damping timescales, which are most important among the damping mechanisms of the r -modes, are almost the same for the two models. The huge difference in the damping timescales associated with the bulk viscosity between the two models may come from the difference in the treatment of the modes in the crust. Although we count the damping contributions in the crust for the model NS13T8, we do not count them for the polytropic model because we have assumed that the crust is static and allows no oscillations. In addition to this, the model NS13T8 in an evolutionary sequence of cooling neutron stars has the crustal temperature that is higher by a factor 7 than the core temperature. Since the bulk viscosity coefficient is proportional to T^6 , the contribution of the bulk viscosity in the crust is largely enhanced for the damping of the r -modes.

4. Discussion and Conclusion

It is well known that retrograde oscillations whose frequency ω satisfies the condition $0 < \omega/\Omega < m$ are unstable to the gravitational radiation reaction (Friedman & Schutz 1978). Using equation (2), we can show that the retrograde crustal toroidal modes satisfy this condition and become unstable to the gravitational radiation reaction when

$$\Omega > \frac{l' (l' + 1)}{m (l'^2 + l' - 1)} \omega_t(0), \quad (8)$$

where $\omega_t(0)$ denotes the oscillation frequency of the mode at $\Omega = 0$. This instability can happen at slow rotation rates of the star if $\omega_t(0)$ is sufficiently small. Note that all crustal toroidal modes shown in Figure 1 are unstable to the gravitational radiation reaction. Our preliminary stability calculation, however, shows that the instability of the crustal toroidal mode is very weak and may not be important.

In this paper, we have investigated the properties of r -modes of slowly rotating neutron stars with a solid crust. We have found that the mode property of the core r_0 mode with $l' = m = 2$, which is most important for the r -mode instability of neutron stars, is strongly affected by mode coupling with the crustal ${}_2t_0$ modes as Ω increases. This means that we cannot assume a simple mode property of the core r_0 modes with $l' = m$ as a function of $\bar{\Omega}$. To discuss the r -mode instability of rotating neutron stars it has been common to derive an extrapolation formula for the damping timescale of the r -modes (e.g., Lindblom et al. 1998). The extrapolation is usually carried out about the interior temperature T and the rotation frequency Ω . The extrapolation with respect to the temperature may be justified if the stars have an isothermal structure characterized, for example, by the central temperature T_c . On the other hand, since the mode properties of the r -mode are well known only in the limit of $\bar{\Omega} \rightarrow 0$, the damping timescales are calculated in this limit and are extrapolated to rapidly rotating neutron stars with $\bar{\Omega} \sim 1$. This process of the extrapolation with respect to Ω is justified if the mode frequency ω is well approximated by $2m\Omega/l'(l' + 1)$ and the eigenfunction has the dominant toroidal component, independent of Ω . We know that the above two assumptions are satisfied for the fundamental r -modes with $l' = m$ for fluid stars (Yoshida & Lee 2000b; Yoshida et al. 2000; Karino et al. 2000). In the case of neutron stars with a solid crust, however, the two assumptions are not satisfied except when $\bar{\Omega} \sim 0$. This means that the extrapolation formula of the damping timescales of the r -modes, derived in the limit of $\bar{\Omega} \rightarrow 0$, is not applicable to the discussion on the r -mode instability of the rapidly rotating stars with a solid crust. In this sense, the discussions based on the extrapolation formula of the damping timescales are not fully justified.

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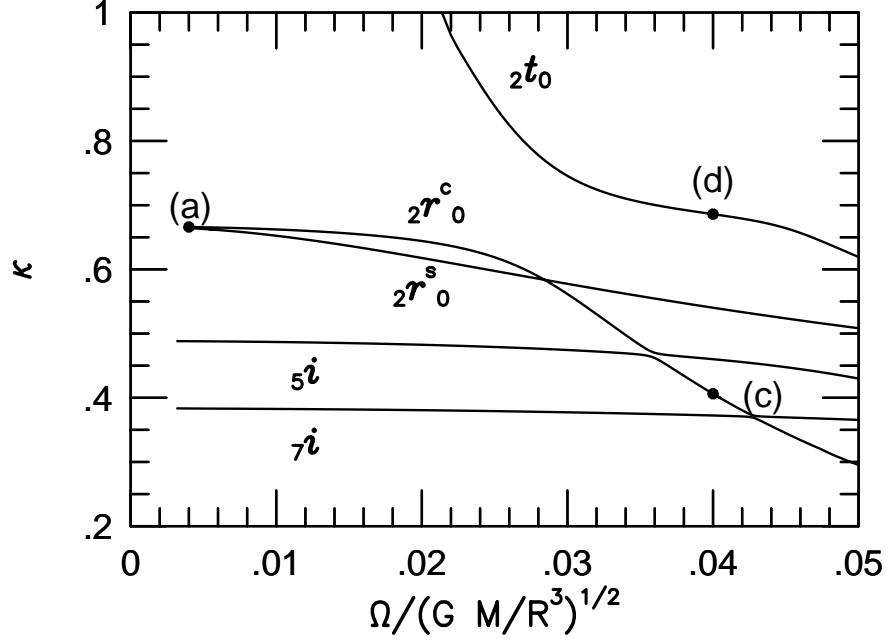


Fig. 1.— Scaled frequencies $\kappa = \omega/\Omega$ of the r -modes, inertial modes and the crustal torsional (toroidal) mode of the model NS13T8 are plotted as functions of $\bar{\Omega} = \Omega/(GM/R^3)^{1/2}$ for $m = 2$, where the frequency curves are attached by the labels indicating their classification at small values of $\bar{\Omega}$ (see the text for the detail of the classification). The modes depicted are retrograde odd modes. The r -modes and the crustal toroidal mode have the dominant toroidal component associated with $l' = m = 2$ at small values of $\bar{\Omega}$. Note that the eigenvalue of the r -modes is given by $\kappa = 2m/(l'(l' + 1))$ in the limit of $\bar{\Omega} \rightarrow 0$. The filled circles indicate the modes whose eigenfunctions are shown in Figure 2.

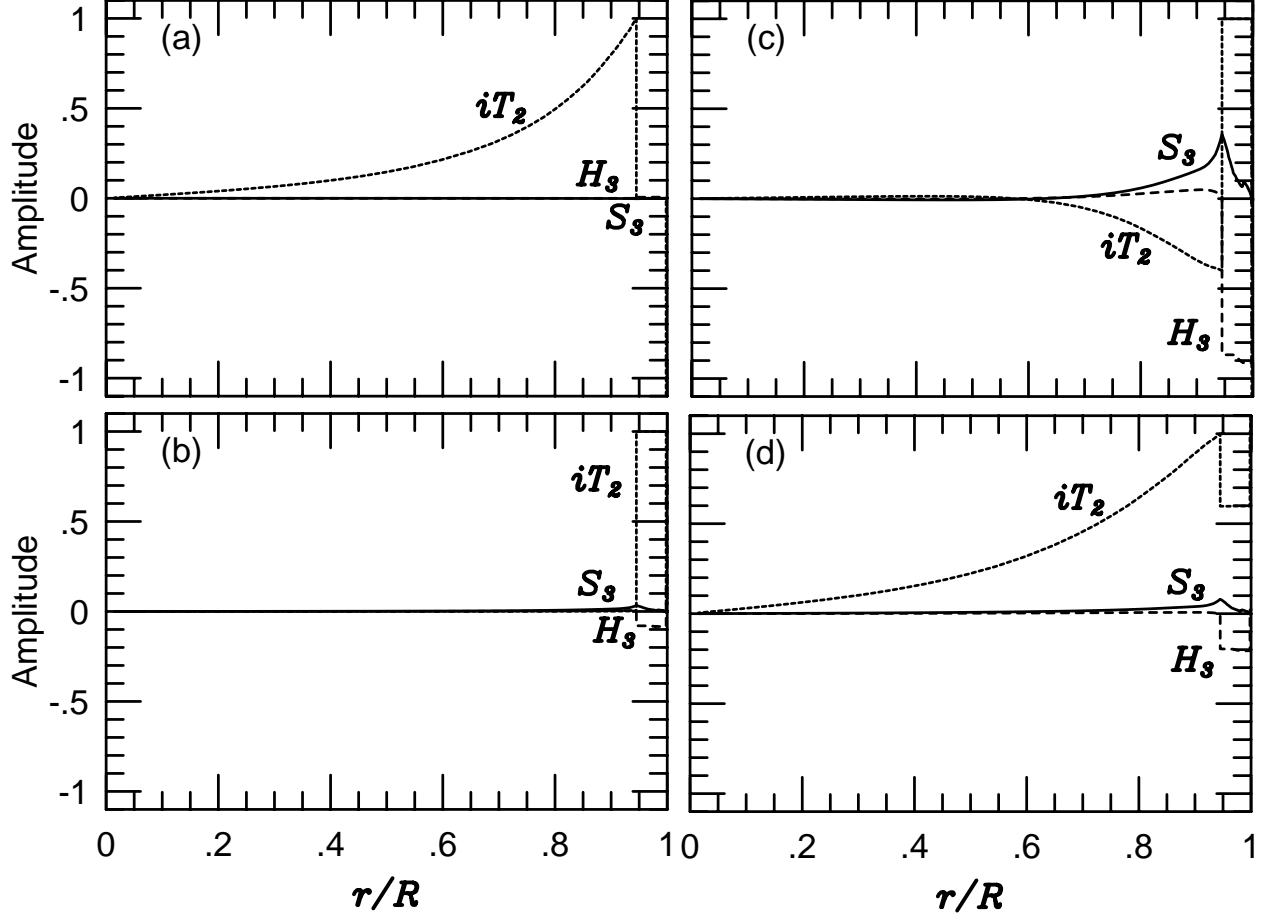


Fig. 2.— Expansion coefficients S_3 (solid curve), H_3 (dashed curve), and iT_2 (dotted curve) of the core r -modes and the crustal toroidal modes with $l' = m = 2$ for the model NS13T8 are given as functions of r/R at $\bar{\Omega} = 4 \times 10^{-3}$ in panels (a) and (b), and at $\bar{\Omega} = 4 \times 10^{-2}$ in panels (c) and (d), where the normalization of the eigenfunctions are given as $iT_2(r_{bc}) = 1$ with r_{bc} being the bottom of the solid crust. The eigenfunctions shown in panels (a) and (b) are those of the ${}_2r_0^c$ -mode and the ${}_2t_0$ -mode, respectively. The locations of the modes are shown in the $\kappa - \bar{\Omega}$ plane by the filled circles in Figure 1.

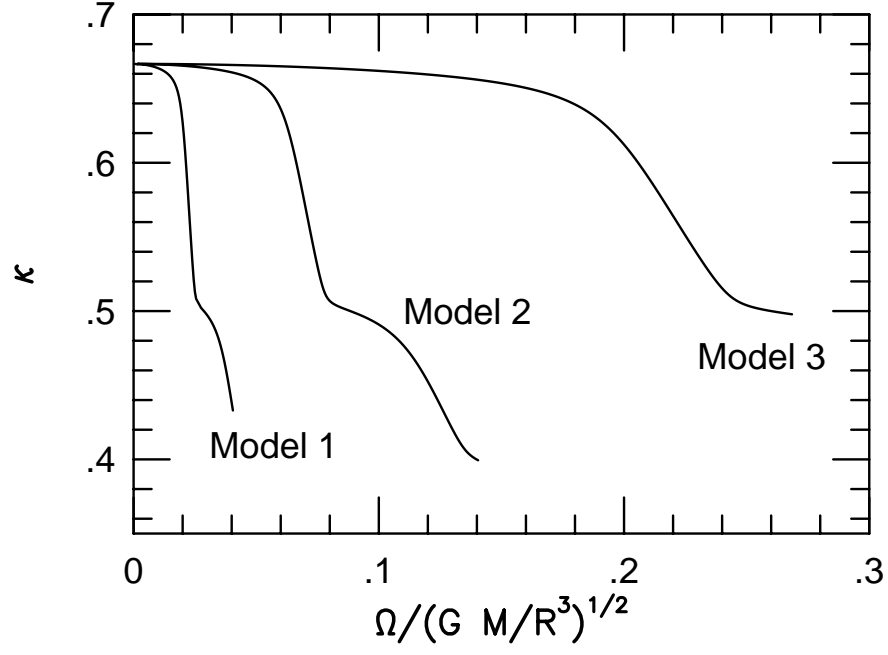


Fig. 3.— Scaled frequencies $\kappa = \omega/\Omega$ for the fundamental ${}_2r_0^c$ modes with $l' = m = 2$ are plotted as functions of $\bar{\Omega} = \Omega/(GM/R^3)^{1/2}$ for the simple polytropic model with a solid crust. The three models have different values of $\mu_0/\rho^{4/3}$ that is assumed to be constant in the crust (see, Table 1).

Table 1. Shear modulus μ_0 and eigenfrequency $\omega_t(0)$ of the ${}_2t_0$ mode for the polytropic model with a crust at $\Omega = 0$.

Model	$\mu_0/\rho^{4/3}$	$\omega_t(0)/(GM/R^3)^{1/2}$
model 1	1.0×10^{11}	9.9×10^{-3}
model 2	1.0×10^{12}	3.1×10^{-2}
model 3	1.0×10^{13}	9.9×10^{-2}

Table 2. Dissipative timescales $\tilde{\tau}$ of core r_0 modes with $l' = m = 2$ for neutron star models.

Model	$\tilde{\tau}_B(\text{s})$	$\tilde{\tau}_S(\text{s})$	$\tilde{\tau}_{VBL}(\text{s})$	$\tilde{\tau}_{J_z m }(\text{s})$
NS13T8	1.3×10^7	1.6×10^5	3.7×10	-2.2×10^{-1}
Polytrope with a static crust	1.7×10^{17}	1.9×10^6	8.9×10	-4.4×10^0